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The mixing Behaviour of Laminar Convection flow in Warm Bathing Water: An In-depth Review

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Abstract: The mixing behaviour of laminar convection flow have been studied taken density as a quadratic function of temperature. There was mixing immediately after both hot and cold water come in contact. The entire contact layer was active with mixing even as the hot water continue to deplete further reducing in volume. The entire fluid in the container became warm without any external force. Profiles of temperature, vertical and horizontal component velocities were also examined at some point (X, 69) below the point where the two fluids meet. Result here have shown that different types of mixed fluid were found at that level in the temperature profile, confirming the fact that there was mixing in that region. Meanwhile, the vertical velocity profile agrees to the fact that there was an insignificant slightly warm fluid in that region as also evident in the various simulations. The downward curves indicates descending fluid while, the upward curves indicates fluid that were still slightly positively buoyant. The horizontal velocity profile showed that there was a fluid motion from left to right and right to the left. This behaviour in the fluid movement is reasonable as descending fluid in vortex form continue to sink, interacting with the ambient fluid. The hot water at the upper section is expected to deplete completely: but then, our result did not capture this behaviour because of the limited simulation time that we have used: and this we regard as a limitation here in this study. The results as presented here are very good as they were able to capture the real flow scenario that give us a better understanding into the mixing behaviour in a convection flow through our daily warm bath.

Keywords: *Convection flow, Ambient fluid, Temperature of maximum density Tm, Laminar.*

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Introduction

Free or natural convection flows are special kind of flows that are buoyancy driven heat transfer systems that occurs in fluids. These flows are evident in our surroundings and as well governed by the temperature gradient. (i.e., fluid motion is as a result of density difference owing to temperature gradient without any external perturbation)(Ezan & Kalfa, 2017). In a more practical sense, water that surrounds any heat source or point may eventually gain heat and become less dense than it was and begin to flow upward, forming buoyancy flow. Once this happens, the surrounding cold fluid will replace the initial cold water that have moved upward and become hot again depending on the heat strength and this will induce the entire surrounding water forming convection flow (Ongodiebi & George, 2025; Kane, 2017; Nayak et al. 2018). When simulating natural convection flows, it is worth noting that the heat transfer rate is dependent on both the geometry and temperature variation in the ambient fluid. In addition, the fluid velocity is also usually small: therefore, the choice of flow parameters used is also very important as it determine the type of flow scenario. The study of natural convection flows have received very wide attention because of its importance in many technological, scientific and engineering applications. And such flows can be observed in solar ponds, cooking, over-head water tanks, ventilation of buildings, Cooling of power plants and electronic equipment, etc., (Ongodiebi & George, 2025; Khurshid & Silaipillayarputhur, 2018; Radhwan & Zaki 2000; Djoubeir et al. 2014).

In the past, Ezan & Kalfa, (2017) have studied natural convection of water near 4°C inside partially heated and cooled ver- tical walls and lay emphasis on the interaction of temperature and velocity distributions by different boundary conditions.

The author observed that convection flow in water become very complex in cold storage applications when the temperature at this point become lower than the density inversion temperature. They also observed that most of the models developed for the simulation of the cold storage units affects the heat transfer system into conduction mode when simplifying the complexity of the equations. But concluded that their result can be helpful to both researchers and designers working on cold storage applications with water as a storage medium. Hossain & Rees, (2005) in their work entitled "Natural Convection Flow of Water near its Density Maximum in a Rectangular Enclosure having Isothermal Walls with Heat Generation" concluded that the flow and temperature field depend very strongly on the internal heat generation parameter and the difference between initial temperature and the mean temperature of the sidewalls. That the flow become symmetric when both the mean and the initial temperature are the same. But then, this behaviour changes when either the mean temperature or internal heat generation parameter varies. More recently, Ongodiebi & George, 2025 have also studied the mixing behaviour of free convection flow in warm bath with the assumption that density was taken as a quadratic function of temperature. Their result showed that mixed dense fluid that have attained the temperature of maximum density could descend from the contact layer in the form of an inverted mushroom like structure towards the floor of the container. Both vertical and horizontal velocity profiles were also considered at some point below the contact layer which also agrees with real scenario. The behaviours as recorded by these authors were also found in the literature by Cianfrini et al. 2015 with a similar configuration. The following authors have also considered some of these flows with different configuration and can be studied for more insight (Li et al. 2011; Rahman et al. 2010; Kane, 2017; Hidayathulla Khan et al. 2018; El Moutaouakil et al. 2020; Cianfrini et al. 2015; Hasnaoui et al. 1992; Lee & Ha, 2006; Zheng et al. 2021; etc.).

It is worth stating here that the findings of Ongodiebi & George, (2025) appears very reasonable and realistic with the choice of flow parameters used for a laminar flow case scenario but then, they have not been able to give a detailed description of the temperature field (temperature distribution) which is a prime factor in convection flows. As highlighted above, natural convection flows are buoyancy driven heat transfer systems that occurs in fluids without any external per- turbation. Thus, the consideration of temperature field is paramount. Therefore we have decided to carryout a numerical simulation with the same configuration and assumptions as also recorded in the literature by Ongodiebi & George, (2025). All flow parameters are the same as those by Ongodiebi & George, (2025) except for the Reynold's number that will be taken to be Re = 50 for a laminar flow scenario. This investigation will enable us to have more insight into the mixing behaviour of both the hot and cold water as they come in contact. The key behaviour to expect is a natural convection flow as bot fluid mixes further.

Model Formulation and Governing Equations

The behaviour of laminar convection flow as both cold and hot water come in contact is of interest. With this it is obvious that after mixing, the more dense fluid will form a descending plume due to the nonlinear relation between density ρ and temperature T is of interest. Therefore, we are proposing a quadratic dependence relation assumption of density on temperature and believe that it is appropriate for this present investigation.

$$\rho = \rho_m - \beta\,(T - T_m)^2. \tag{1}$$

The quadratic dependence relation assumption have shown to gives a good fit to experimentally determined density in fresh water at temperatures below 10◦C, taken Tm = 3.98◦C, ρm = 1.000 × 103 kg.m−3 and β = 8.0 × 10−3 kg.m−3(◦C)−2 (Moore & Weiss, 1973; Oosthuizen &Paul, 1996) and all other fluid properties such as viscosity, thermal diffusivity are assumed constant. We assume that the flow is two dimensional and time dependent with liquid property being constant except for the water density, which changes with temperature. We can non-dimensionalise the coordinates x, y, velocity components u, v, time t, pressure p and temperature T by.

$$U = \frac{u}{U_*} \quad V = \frac{v}{U_*} \quad X = \frac{x}{H} \quad Y = \frac{y}{H} \quad \tau = \frac{t}{\frac{H}{U_*}} \quad P = \frac{p}{\rho U_*^2}$$

$$\varphi = \frac{T - T_\infty}{T_m - T_\infty} \tag{2}$$

Where x and u are horizontal, y and v are vertical $U_* = \sqrt{\frac{\rho_\infty - \rho}{\rho} H}$ is the relative frontal velocity and domain height H. We also define dimensionless parameters, the Reynolds Re, Prandtl Pr and Froude Fr numbers, by.

$$\nu = \frac{\mu}{\rho} \quad \alpha = \frac{k}{\rho c_p} \quad Re = \frac{U_* H}{\nu} \quad Pr = \frac{\nu}{\alpha} \quad Fr^2 = \frac{\rho_m U_*^2}{g\beta(T_m - T_\infty)^2 H}, \tag{3}$$

Where ν and α are the respective diffusivities of momentum and heat, and μ is viscosity, k is thermal conductivity and cp is specific heat capacity. With the dimensionless variables and parameters, the continuity equation, horizontal and vertical momentum equations and thermal energy equation are given as.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{4}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{5}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{1}{Fr^2}[\phi^2 - 2\phi] \tag{6}$$

$$\frac{\partial \phi}{\partial \tau} + U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} = \frac{1}{RePr}\left(\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2}\right) \tag{7}$$

Our computational domain is also consists of a domain length L of total L = 70, i.e., $0 \leq X \leq 70$, and a domain height H = 90 i.e., $0 \leq Y \leq 90$. Where the domain length and height of the hot upper section is L1 = 70 and H1 = 10 respectively: while, the ambient fluid domain length and height is L = 70 and H = 80. All side walls and the horizontal base of the container are considered insulated with a surface condition that is considered adiabatic.

Therefore, our initial conditions are an undisturbed, homogeneous medium.

$$U = 0, \quad V = 0, \quad \phi = 0, \quad for\ the\ cold\ section \quad \tau < 0 \tag{8}$$

And

$$U = 0, \quad V = 0, \quad \phi = 2.5, \quad for\ the\ hot\ section \quad \tau < 0 \tag{9}$$

For $\tau \geq 0$ we have boundary conditions as follows. On the side walls:

$$U = 0, \quad V = 0, \quad \frac{\partial \phi}{\partial X} = 0 \tag{10}$$

At the interaction layer (source):

$$U = 0, \quad V(X,0) = 1, \quad \phi = 2.5 \text{ for } L_1 \text{ and } \phi = 0 \text{ for } L,$$
$$\text{for } X = 70,\ at\ Y = H_1\ and\ H\ respectively. \tag{11}$$

On the floor of the domain:

$$U = 0, \quad V = 0, \quad \frac{\partial \phi}{\partial Y} = 0 \tag{12}$$

At the top of the domain:

$$\frac{\partial U}{\partial Y} = 0, \quad V = 0, \quad \frac{\partial \phi}{\partial Y} = 0 \tag{13}$$

Taken Reynolds number Re = 50, Froude number F r = 2.5 and Prandtl number P r = 9.5 all will be kept fixed throughout the investigation. The dimensionless temperature φ = 2.5 in the L1 is equivalent to a temperature at 10◦C placed at the surface of an ambient temperature 0 ◦C. Result is by means of COMSOL Multiphysics software. It is commercial software that uses the finite element solver with discretization by the Galerkin method and stabilisation to prevent spurious oscillations. Our results will be independent of the mesh used, if only the mesh size is ≤ 0.05. More information about the numerical methods is available from the COMSOL Multiphysics website (COMSOL Multiphysics Cyclopedia, 2016). Results will be presentation will by surface temperature plots of dimensionless temperature on a cooler scale from dark red for the ambient temperature φ = 0.0, through yellow to white for the source temperature φ = 2.5. Note that φ = 1.0 corresponds to the temperature of maximum density while φ = 2.0 is the temperature at which warm water has the same density as the ambient cold water.

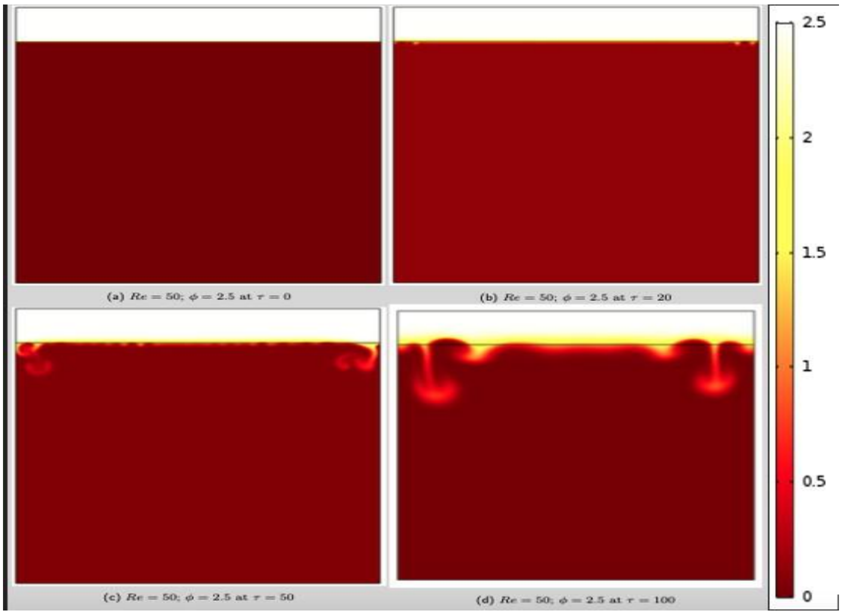

***Fig. 1:** Evolution of temperature field in convective flow for Re = 50, F r = 2.5 and Prandtl number P r = 9.5 and dimensionless temperature φ = 2.5 at the upper section within the time range 0 ≤ τ ≤ 100.*

Numerical Results

The mixing behavior of fluid with different densities had just been investigated, taken density as a quadratic function of temperature. As highlighted above, Re = 50, F r = 2.5 and P r = 9.5 are kept fixed throughout the study for a laminar flow case. Result in Figure 1 (a) & (b) shows the development of temperature flied as both hot and cold water come in contact. An increase in the dimensionless time showed that interaction between the two fluid had produced some dense fluid in the contact layer that descends forming a descending plume to the floor of the container. It is very obvious that most of the fluid that descends have already attained the temperature of maximum density or a temperature very close to it (see Fig. 1(d)). It is also interesting to note that the descending dense but warm fluid continue to interact, inducing the surrounding water further. The result showed that mixing takes place in the entire contact layer even as the volume of the hot water continue to deplete further through mixing: while the surrounding water continue to gain more heat through convection (see Fig. 2 (e), (f), (g) & (h)). This process will continue until the entire hot water at the upper section will get mixed up with the ambient cold fluid and the entire fluid in the container will become the same temperature without any external mixing force. Investigation using this configuration have received less attention in the past where most convection flow configurations considered their heat activation point on the vertical walls or both the vertical and the horizontal walls. But then, the investigations by Cianfrini et al. (2015) and Ongodiebi & George, (2025) have similar configuration and their results also appears very similar as compared to our present results, especially those by Ongodiebi & George, (2025) with the quadratic relation assumption. Only that significant dense fluid could be observed at earlier time in this present study as compared to those by Ongodiebi & George, (2025) which might be as a result of the slight increase in the Reynolds number (Re = 50) here in this study. This indicates that there has been a slight increase in the mixing rate which in turn resulted to a quick production of dense fluid that descends to the floor. Meanwhile, Cianfrini et al. (2015) with the linear dependent relation assumption have reported that by fixing.

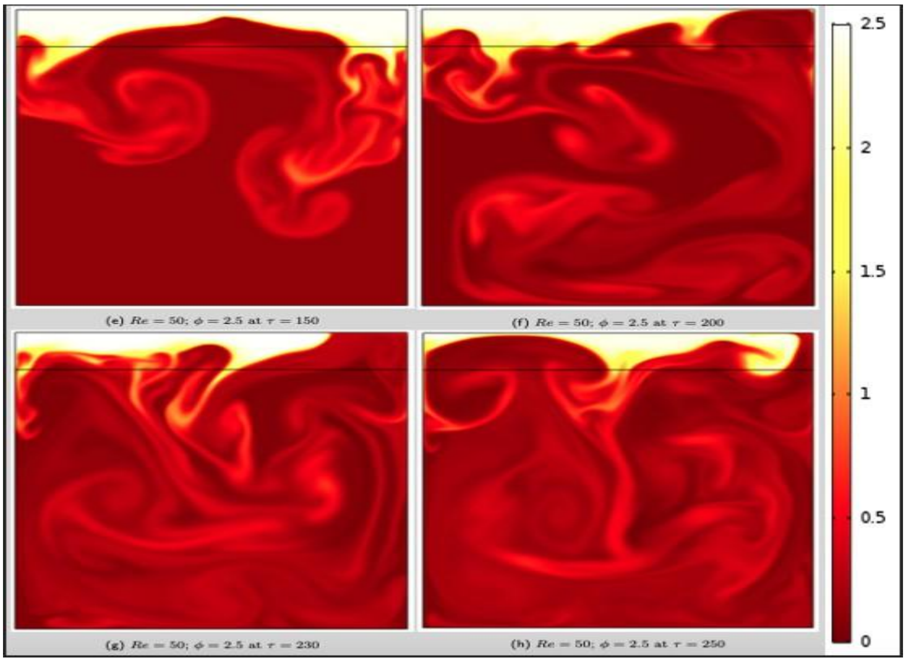

Fig. 2: Evolution of temperature field in convective flow for Re = 5, F r = 2.5 and Prandtl number P r = 9.5 and dimensionless temperature φ = 2.5 at the upper section within the time range 150 ≤ τ ≤ 250.

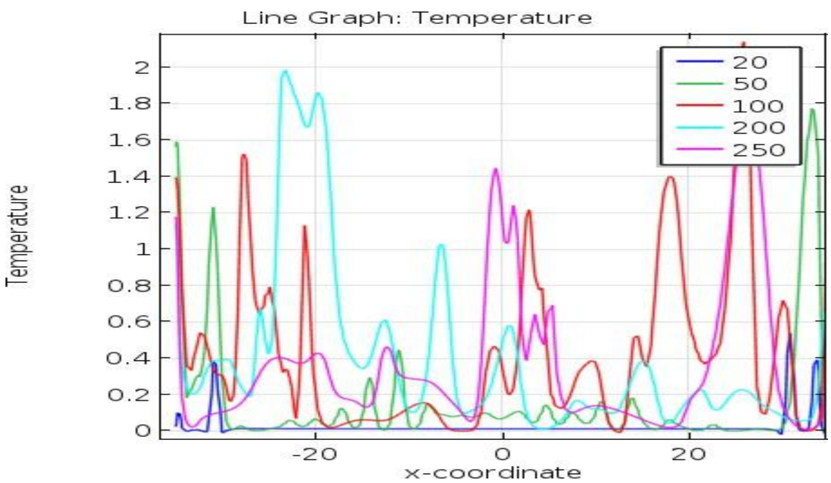

Fig. 3: Dimensionless Temperature profiles at some point close to the mixing layer T (X, 69) at time τ = 20, 50, 100, 200, 250.

The temperature of the cooled wall, penetrative convection will take place owing that the temperature of the upper heated surface is lower than a limit value that increases with increasing the cavity width. Though, we did not given any discussion on penetrative convection because our domain of configuration and temperature of the fluid in the various sections was kept fixed. Profiles of temperature, y-component and x-component velocities were also examined and plotted as a function of the x-coordinate at some point (X, 69) below the contact layer (see figure 3, 4 & 5). Profiles of temperature as shown in Fig. 3 indicates that most of the fluid at that point was already dense within the indicated time interval. This means that mixing commenced immediately the thin layer was removed leading to the observation of slightly warm fluid even at time $\tau = 20$. Meanwhile, at dimensionless time $\tau = 50$ some of the fluid had already attained the temperature of maximum density. And as time progresses, different types of mixed fluid could be found at that level. Thus, the more the interaction between the two fluid, the more the hot water get depleted through mixing inducing the entire surrounding fluid. This fluid movement as the two fluid mixes in all direction enables us to talk about the various velocity profiles. At earlier time interval the vertical velocity profiles agrees to the fact that there was an insignificant slightly warm fluid in that region as also evident in Fig. 1 (b) & (c). But later, significant mixed fluids were observed in the region. As also recorded in the literature by Ongodiebi & George, (2025) downward curves here indicates descending fluid while, the upward curves indicates fluid that were still slightly positively buoyant. The behaviour in the curves at earlier time appears different from those by Ongodiebi & George, (2025) and this is as a result of the slight increase in the Reynolds number resulting to a slightly increase the mixing rate (see Fig. 4). In a similar manner, profiles of x-component velocity were also considered and plotted against the x-coordinate (see figure 5). The result also indicated that there was a fluid motion from left to right and right to the left. This behaviour in the fluid movement is reasonable as descending fluid in vortex form continue to descend, interacting with the ambient fluid. The downward and upward fluctuation in the curves also appears slightly different from those by Ongodiebi & George, (2025) which is also to the fact that there was a slight increase in the mixing rate. It is worth noting here that the hot water at the upper section is expected to deplete (mixed up).

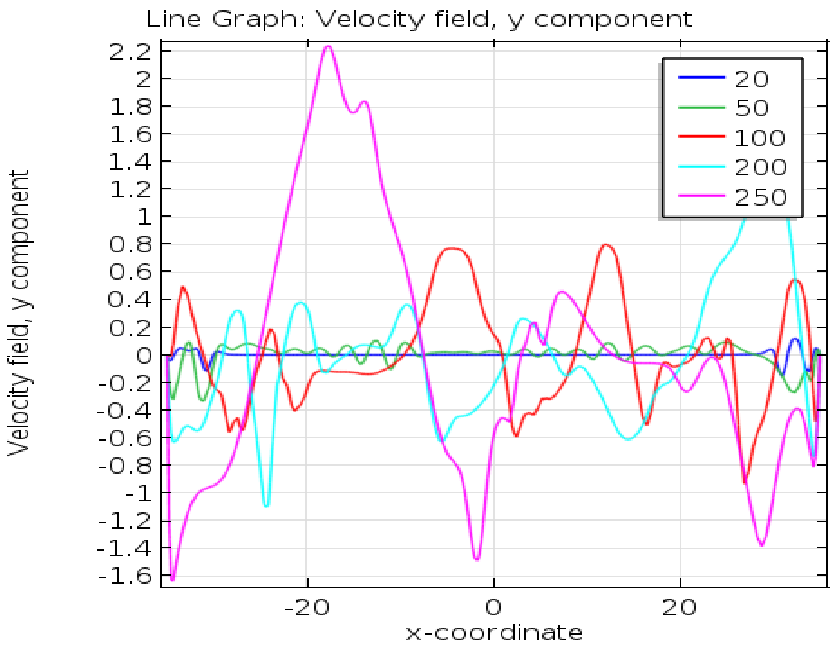

Fig. 4: Dimensionless vertical velocity profiles at some point close to the mixing layer V (X, 69) at time τ = 20, 50, 100, 200, 250.

Completely: but then, our result did not capture this behaviour which is as a result of the limited simulation time that have been considered. Thus, this we regard as a limitation here in this study. The results as presented here are very good as they give capture the real flow scenario that give us a better insight into the mixing behaviour in a convection flow through our daily warm bath. In conclusion, whenever water masses of different densities meet, mixing will take place without any perturbation until the entire surrounding fluid is induced through convection flow.

Discussion/Conclusion

Laminar convection flows and their possible mixing behaviours was investigated with the quadratic dependence relation assumption. In the result, mixing began immediately after hot and cold water come in contact. As time progresses, all mixtures that have become dense at the initial interaction layer descended and as well inducing the surrounding water as it sinks to the floor. The entire contact layer was active with mixing even as the hot water continues to deplete further. It is expected with time that the entire fluid in the container will become the same temperature without any external mixing force as hot water deplete completely. But then, this was not achieved because of the limited simulation time and this we regard as a limitation here in this study.. However, results by Cianfrini et al. (2015) and Ongodiebi & George, (2025) appears very similarities with ours especially, those by Ongodiebi & George, (2025) with the same configuration. The little difference is that, a significant dense fluid could be observed at earlier time interval in this present study as compared to those by Ongodiebi & George, (2025): and this might be as a result of the slight increase in the Reynolds number (Re = 50) that have influence the mixing rate. This is reasonable because slightly intense mixing will result to quick production of dense fluid. Meanwhile, Cianfrini et al. (2015) with a linear dependent relation assumption talked about penetrative convection. That this takes place provided the temperature of the upper heated surface is lower than a limit value that increases with increasing the cavity width. But then, we have not considered penetrative convection because our domain of configuration and temperature of the fluid in the various sections was kept fixed. Profiles of temperature,

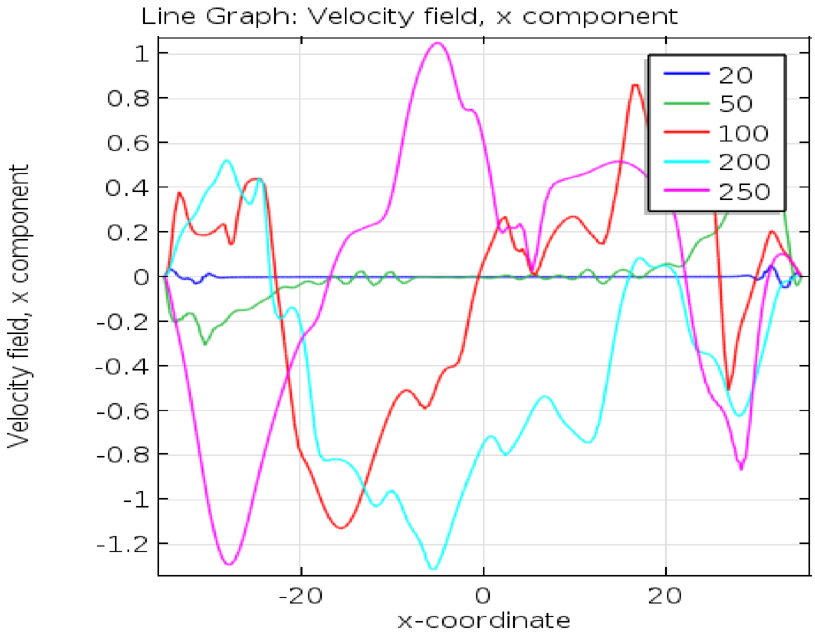

Fig. 5: Dimensionless horizontal velocity profiles at some point close to the mixing layer V (X, 69) at time τ = 20, 50, 100, 200, 250.

Vertical and horizontal component velocities were also examined at some point (X, 69) below the contact layer. Where results in the temperature profile showed that different degree of mixed fluid were found at that level, confirming the fact that there was mixing in that region. Meanwhile, the vertical velocity profiles agrees to the fact that there was an insignificant slightly warm but dense fluid in that region that descends as also evident in Fig. 1 (b) & (c). Note here that the downward curves indicates descending fluid while, the upward curves indicates fluid that were still slightly positively buoyant. The behaviour in the curves at earlier time appears different from those by Ongodiebi & George, (2025) and this is as a result of the slight increase in Reynolds number which had resulted in the quick production of dense fluid in the present study. The horizontal velocity profile indicated that there was a fluid motion from left to right and right to the left. This behaviour in the fluid movement is reasonable as descending fluid in vortex form continue to sink, interacting with the ambient fluid. The results as presented here are very good as they give capture the real flow scenario that give us a better understanding into the mixing behaviour in a convection flow through our daily warm bath.

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